# Model of heterogeneous interactions among complex agents. From a neural to a social network

**L. Yu. Zhilyakova**

V. A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences
65, Profsoyuznaya street, Moscow 117997, Russia

**Abstract.** We describe a heterogeneous neural network where neurons interact by means various neurotransmitters. This feature enables exerting selective influence on neurons. We use this formalism as a basis for modeling interactions between agents in a social network in which two types of opposite activity are spreading. The main properties of agents and principles of activity spreading are defined. The classification of agents according to their parameters is represented.

**Keywords:** discrete dynamics, heterogeneous neural network, social network, activity in networks

## *Introduction*

We describe a threshold network model in which nodes exchange several kinds of different transmitters. The network operates in discrete time. At each time step, active nodes secret a certain amount of defined transmitter. At the next step, these transmitters become available to other nodes if they have appropriate receptors. Every node has its own set of receptors. The activity of neurons at time step $t$ is determined by input signals and its internal state. This state is characterized by a single generalized parameter, which can be called *readiness for activation*.

A large number of studies are devoted to different dynamic network models. However, despite the diversity of approaches and the wide range of simulated tasks, as a rule, these models consider the propagation of one type of activity. A wide class of such models is described by random walks or diffusion on graphs (see, for example, survey papers [1, 2]). A completely different kind of dynamics is described in the integer threshold model chip-firing game. This model on oriented and undirected graphs is well studied and described analytically [2–4]. Such threshold models, in particular, describe phenomena of self-organized criticality "avalanche" or "sand- (rice-) pile" [5, 6]. Basically, these models consider the series of consecutive firing, when the vertices fire in turn in random order. It was proved that if the final configuration exists, it does not depend on the shots order. Therefore, such models are often called "*abelian* sandpiles" [6].

The study of social networks is a rapidly developing area. Numerous researchers have obtained many interesting results on the general regularities in the structure of social networks (small-world property, power-law distribution of degrees, modularity, the phenomenon of the rich club, assortativity, etc.), and on the various dynamic processes occurring in these networks [7].

In the field of modeling the activity spreading processes in social networks, various mathematical models are being developed: threshold and cascade models [8–10], models of epidemic spreading [11], Markovian models [12], and a number of others. Based on these models, optimization problems are solved, in particular, the determination of the initial set of active agents that ensure the maximum propagation of activity over the network [8, 13]. Models of social networks control were proposed and developed in a number of works (see e.g. [14]).

In this article, we use a model of multitransmitter neuronal interaction [15–17] for simulating the spreading of two types of opposite activities in a social network. Some approaches to this problem were described in [18] and then developed in [19].

## Model description

We consider heterogeneous neural network $\mathbf{S} = \langle N, E, C \rangle$, where $N = \{1, \ldots, n\}$ is the set of neurons, $E$ is the set of connections between the neurons, $C = \{c_1, \ldots, c_m\}$ is the set of transmitters. The network operates in discrete time $T$. The rules of the interaction of neurons are as follows. If the neuron $i$ emits the transmitter $c_k$, and the neuron $j$ has a receptor to this transmitter, there is an oriented connection between the neurons. We assume that each transmitter has its own color. Then connections $E$ will be colored: $E = E_{c_1} \cup \ldots \cup E_{c_m}$, and their weights $r_{ijc_k}$ correspond to the sensitivity of receptors of the receiving neurons. We will make no distinction between synaptic and extrasynaptic connections.

When activated, every neuron transmits a specific transmitter over all its outgoing connections. We assume that a neuron can always produce a sufficient amount of transmitter, and each outgoing edge receives amount of transmitter equal to its weight $r_{ijc_k}$.

Another characteristic of a neuron is the membrane potential (MP) $U_i(t)$. In the model it is believed that the higher the membrane potential of the neuron, the easier it becomes excited, and vice versa. The neuron $i$ becomes active if its membrane potential $U_i(t)$ is above a certain threshold value $P_i$.

The membrane potential of neurons at the time $t$ is calculated by the formula

$$U_i(t) = \alpha \cdot U_i(t-1) + \beta \cdot \sum_{j=1}^{n} \sum_{k=1}^{m} r_{jic_k} y_j(t-1)\,, \qquad (1)$$

where $\alpha \in (0\,,\,1]$ is a discount factor, $\beta$ is a scale factor.

The activity function $y_i(t)$ of a neuron $i$ at time $t$ is determined by the formula

$$y_i(t) = \begin{cases} 1, \text{if } U_i(t) \geq P_i; \\ 0 \text{ otherwise.} \end{cases} \qquad (2)$$

## *Model of a social network with five types of agents*

Using a specified formalism we construct a model of social network **S** = <*N*, *E*, *C*>, where $N = \{1, \dots, n\}$ is the set of agents, $C = \{c_1, \dots, c_m\}$ is the set of types of their activities, $E = E_{c_1} \cup ... \cup E_{c_m}$ is the set of colored weighted edges. There may be several edges of different colors and weights between a pair of nodes $i$, $j$; their weights indicate the degree of influence. The only membrane potential in this case is replaced by $m$ parameters $U_{ic_k}(t)$ corresponding to the agent's readiness to be activated by one of the $m$ types. Accordingly, each agent $i$ has $m$ thresholds $P_{ic_k}$.

Here we consider a reduced case with $m = 2$ and assume that there are two activities in a network (called 1 and 2). In this case, each agent will have two parameters $U_{i1,2}(t)$ – one for each activity type, and two threshold values $P_{i1,2}$.

Types 1 and 2 denote antagonistic activities (revolutionary/reactionary, constructive/destructive, sharp/blunt end of the egg should be cracked etc.).

At each time step $t$ agents receive from their neighborhood the activity of one or two types. These signals effect to non-constant parameters of agents by the formulas similar to (1)–(2). Except that the second formula compares absolute values, because for some agents types of activity have different signs:

$$y_{i1,2}(t) = \begin{cases} 1, \text{ if } \left|U_{i1,2}(t)\right| \geq \left|P_{i1,2}\right|; \\ 0 \text{ otherwise.} \end{cases}$$

### States of agents and their mutual influence

Agents in the network have different thresholds for the two types of activity. The thresholds are the normalized quantities: $P_{i1}, P_{i2} \in [-1, 1]$. Signs of $P_{i1}$ and $P_{i2}$ determine the agent's attitude to these types of activity (positive or negative). $|P_{i1}|$ and $|P_{i2}|$ indicate the weighted fraction of the active neighbors from all neighbors influencing the agent. Accordingly, the coefficient $\beta$ in formula (1) is inversely proportional to the indegree of vertex $i$ in the graph $<N, E>$. The signs of values $r_{ji1}$ and $r_{ji2}$ always coincide with the signs $P_{i1}$ and $P_{i2}$ respectively. The agents' attitudes to the two kinds of activity can be combined in different ways. Table 1 shows all possible combinations.

**Table 1.** Possible combinations of threshold signs for an individual agent

| | $P_{i1}$ | $P_{i2}$ |
|---|---|---|
| 1. | $> 0$ | $< 0$ |
| 2. | $< 0$ | $> 0$ |
| 3. | $> 0$ | $> 0$ |

In cases 1 and 2, agents have moral certainty about types of activity, one of which is regarded as positive, the other as negative. The activity with the "+" sign will be called *own* for the agent. Whichever threshold ($P_{i1}$ or $P_{i2}$) is exceeded, the agents activate only in *their own* type.

Case 3 describes the situation when agents don't have a clear position for these types of activity. However, the agent $i$ can activate if one of thresholds $P_{i1}$ or $P_{i2}$ is exceeded. In this case, agent $i$ activates by the type of the threshold exceeded. It means that agent succumbs to the charm of the crowd and acts with the majority. If two thresholds were exceeded the same time, the agent selects as *own* one of two activities with equal probability.

The case $P_{i1}$, $P_{i2} < 0$ is not considered. Such an agent cannot start acting, because none of types can be *own*.

## Interpretation of parameter values

The values of thresholds have the obvious interpretation. Let some agent $i$ has threshold values: $P_{i1} = 0.2$, $P_{i2} = -0.99$. Different signs of thresholds mean that the agent has some beliefs, and the first activity is *his own*. Absolute threshold values show the agent's tolerance to the activity of his environment. The higher values $|P_{i1}|$ and $|P_{i2}|$, the more stable is the agent to external activity. Thus, agent $i$ easily activates in a friendly environment, and almost never activates in an environment with beliefs alien to him.

On the contrary, the set of parameters $P_{i1} = 0.99$, $P_{i2} = -0.2$ characterizes the agent with a strong sense of contradiction. If the "own" type of activity prevails, he is silent, because everything is good without his participation. But if the activity is alien, he activates to resist the majority. If both thresholds of the agent are low, this means that he is easily activated by any external activity. The high thresholds characterize cautious agents. Recall that whatever threshold is reached first, the agent for whom $P_{i1} \cdot P_{i2} < 0$ is always activated by its *own* type. At the next time step after activation all agent parameters are reset: $U_{i1,2}(t+1)=0$.

### Types of agents

Each agent belongs to one of the five disjoint classes depending on the values of the parameters $P_{i1}$, $P_{i2}$:
$V = V_{R1} \cup V_{R2} \cup V_A \cup V_C \cup V_P$, where
$V_{R1}$ – revolutionary agents;
$V_{R2}$ – reactionary agents;
$V_A$ – precautionary agents;
$V_C$ – conformist agents;
$V_P$ – passive agents.
**Revolutionaries and reactionaries**. The number of agents in the classes $V_{R1}$ and $V_{R2}$ is relatively small. These passionary agents have almost zero thresholds. For *revolutionary agents* we have $P_{i1} = 0, P_{i2} = -\varepsilon$, for *reactionary agents* $P_{i1} = -\varepsilon$, $P_{i2} = \varepsilon$. These agents have no memory and become active as soon as activity of any type comes to them. Revolutionaries differ from reactionaries only by their ability to start the new activity. The reactionaries can only respond to the activity of other network agents.
**Precautionary agents**, as well as agents of the first two types, have their own beliefs, that is, $P_{i1} \cdot P_{i2} < 0$. However, they tend to act cautiously, and their values of $P_{i1}$ and $P_{i2}$ vary in the intervals (0, 1), (–1, 0) without taking extreme values.
**Conformist agents** do not give preference to the kind of activity: $P_{i1}$, $P_{i2} > 0$. At small values of $P_{i1}$ and $P_{i2}$ we have "hooligan agents" supporting any activity for the sake of activity itself. At $P_{i1}$, $P_{i2}$, close to 1, the agent activates, yielding to the influence of a large crowd.
**Passive agents** always have $P_{i1}$, $P_{i2} > 1$, and no activity of neighbors is able to draw them into activity.

## *Conclusion*

The paper presents a model of a network of heterogeneous neurons interacting through the common extracellular space via different neurotransmitters. The basic principles of this model are applied in describing the interactions of agents in social networks. It seems to us that social networks are not the only possible area of use of the proposed apparatus. The basic concepts can be applied in many other areas with common properties: multiproduct traffic, internal structure of nodes, their threshold switching, and variability in behavior.

**Acknowledgments** This work was supported by the Russian Science Foundation, project no. 15-07-02488.

## *References*

1. Blanchard, Ph., Volchenkov, D. Random Walks and Diffusions on Graphs and Databases: An Introduction (Springer Series in Synergetics). Springer-Verlag – Berlin–Heidelberg. 2011.
2. Lovasz L. and Winkler P. Mixing of Random Walks and Other Diffusions on a Graph // Surveys in Combinatorics, 1995 (ed. P. Rowlinson), London Math. Soc. Lecture Notes Series 218, Cambridge Univ. Press, 119–154.
3. Biggs, N.L. Chip-Firing and the Critical Group of a Graph // Journal of Algebraic Combinatorics 9 (1999), pp. 25–45. Kluwer Academic Publishers. Netherlands. 1999.
4. Bjorner A., Lovasz L. Chip-firing games on directed graphs, J. Algebraic Combinatorics 1 (1992), 305–328.
5. Bak, P. How Nature Works: The Science of Self-Organized Criticality. New York: Copernicus. 1996.
6. Dhar, D. The abelian sandpile and related models // Physica A: Statistical Mechanics and its Applications. Volume 263, Issues 1–4, 1 February 1999, pp. 4 – 25.
7. Newman M.E.J. The structure and function of complex networks. SIAM Rev 45(2):167–256. 2003.
8. Kempe D., Kleinberg J., Tardos E. Maximizing the Spread of Influence through a Social Network / Proceedings of the 9-th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. 2003. P. 137-146.
9. Watts D.J. A simple model of global cascade on random networks. Proc Natl Acad Sci USA 99(9):5766–5771. 2002.
10. Goldenberg J., Libai B., Muller E. Talk of the Network: A Complex Systems Look at the Underlying Process of Word-of-Mouth // Marketing Letters. 2001. № 2. P. 11-34.
11. Pastor-Satorras R., Vespignani A. Epidemic Spreading in Scale-Free Networks // Physical Review Letters. 2001. № 14(86). P. 3200-3203.
12. DeGroot M.H. Reaching a consensus // J. Amer. Statist. Assoc. – 1974. – Vol. 69, No. 345. – P. 118–121.
13. Goyal A., Bonchi F., Lakshmanan L.V.S., Venkatasubramanian S. On minimizing budget and time in influence propagation over social networks. Social network analysis and mining, 2(1), 2012.
14. Gubanov D.A., Chkhartishvili A.G. Models of information opinion and trust control of social network members / Proceedings of the 18th IFAC World Congress, 2011 World Congress. Milano: International Federation of Automatic Control (IFAC), 2011. P. 1991-1996.
15. Bargmann C.I. Beyond the connectome: how neuromodulators shape neural circuits // Bioessays. 2012. Jun;34(6):458-65.
16. Dyakonova V.Ye. 2012. Neyrotransmitternyye mekhanizmy kontekst-zavisimogo povedeniya. Zhurn. vyssh. nerv. deyat. 62(6):1–17.
17. Sakharov D.A. 2012. Biologicheskiy substrat generatsii povedencheskikh aktov. Zhurn. obshch. biologii. 73(5):334-348.
18. Zhilyakova L. Yu. Network model of spreading of several activity types among complex agents and ITS applications // Ontology of Designing. 2015. Vol. 5. No. 3(17). P. 278-296. (in Russian)
19. Gubanov D.A., Zhilyakova L. Yu. Ob odnoj porogovoj modeli rasprostranenija aktivnosti v social'noj seti // Materialy 8-th Vserossijskoj multiconferencii po problemam upravlenija – Rostov-na-Donu: izdatelstvo SFedU, 2015. Vol.1. P. 51-53. (in Russian)